**Solving N-queen problem using dynamic programming**

**Introduction**

The n-queens problem is to determine Q(n), the number of different ways in which II queens may be placed on an n-by-n chessboard so that no two queens are on the same row, column or diagonal. The problem has a long history. The g-by-8 case was posed in an anonymous problem in 1848. This was later attributed to Max Bezzel. In the years that followed, the problem has attracted the attention of a number of famous mathematicians, including Gauss, Lucas and Polya. It is of current interest mostly as a benchmark problem for backtracking algorithms, it is representative of a large literature. Yet there appears to be no algorithm whose complexity is known to be better than the brute- force approach. A related problem is the toroidal n-queens problem. This problem is identical to the n-queens problem, except that all diagonals are of length n and wrap as if the chessboard were on a torus. If we denote the number of solutions to the toroidal problem as T(n), it is obvious that T(n) < Q(n). In this paper we describe a simple algorithm for computing Q(n) or T(n) in time O(f(n>8”>, where f(n) is a low-order polynomial. The algorithm is based on dynamic programming. It is a special case of an approach developed in, which views certain search problems as integer linear programming problems and solves them via polynomial multiplication.

**Proof**

Consider the toroidal n-queens problem first. In the first row, the first square contributes four semi-exhausted lines (namely, the four lines passing through it>, and the remaining lines in the row contribute three new-exhausted lines (namely, all the lines passing through each except the row line). The last square in the row exhausts the row, hence deleting one row. So after the first row, there are 3n semi-exhausted lines. Subsequent rows add a new semi-exhausted line with their first square, and exhaust it with their last square. (Of course, the final row behaves differently by exhausting various lines, but that does not affect our worst-case analysis.) The non-toroidal case is very similar. In the first row, the first square adds four new lines, and subsequent squares add three new semi-exhausted lines apiece. The last square in the row exhausts two lines: the row, and one of its diagonals (namely, the short single-element diagonal). So after the first row there are 3n - 1 semi-exhausted lines. The first square in the second row adds a new row line and a new diagonal line, and exhausts the other (two-element) diagonal, for a net gain of one line. The remaining squares in the second row have no effect until we get to the last square, which adds a new diagonal and exhausts the row line and a short (two-element) diagonal, for a net loss of one line. Subsequent rows be- have like the second row. Again, the final row exhausts various lines, but that does not concern us.

By taking advantage of line exhaustion and proceeding in row-major order, we have improved the running time of our algorithm considerably.

**Conclusions**

We have described an 0(f(n>8n) algorithm for the n-queens problem. There is some evidence that the number of solutions to the problem is super-exponential. If this is true, then our algorithm is superior to any approach (such as backtracking) that explicitly constructs all solutions to the problem.

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